

Consistency Relation and Non-Gaussianity in a Galileon Inflation

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Abstract. We study a particular Galileon inflation in the light of Planck2015 observational data in order to constraint the model parameter space. We study the spectrum of the primordial modes of the density perturbations by expanding the action up to the second order in perturbations. Then we pursue by expanding the action up to the third order and find the three point correlation functions to find the amplitude of the non-Gaussianity of the primordial perturbations in this setup. We study the amplitude of the non-Gaussianity both in equilateral and orthogonal configurations and test the model with recent observational data. Our analysis shows that for some ranges of the non-minimal coupling parameter, the model is consistent with observation and it is also possible to have large non-Gaussianity which would be observable by future improvements in experiments. Moreover, we obtain the tilt of the tensor power spectrum and test the standard inflationary consistency relation ($r = -8n_T$) against the latest bounds from the Planck2015 dataset. We find a slight deviation from the standard consistency relation in this setup. Nevertheless, such a deviation seems not to be sufficiently remarkable to be detected confidently.

Keywords: Galileon Inflation, Cosmological Perturbations, Consistency Relation, Non-Gaussianity, Observational Constraints.

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1 Introduction

Theory of cosmological inflation is basically related to a quasi-de Sitter universe, a homogeneous and isotropic universe that expands almost exponentially fast, with a nearly constant event horizon. This paradigm is known as the most successful theory to date that describes primordial density perturbations of the universe. These primordial perturbations are arisen from the quantum behavior of both the spacetime metric and the scalar field which derives inflation [1, 2, 3, 4, 5, 6, 7, 8]. Simple inflationary models predict (for instance, via study of the Cosmic Microwave Background (CMB) radiation) nearly scale invariant, adiabatic and Gaussian perturbations [9, 10, 11, 12, 13]. However, some degree of non-linearities must be present at some level in all inflation models, since non-linearity is the inherent property of gravity in essence and also most of the inflationary scenarios have interacting or self-interacting potentials [14, 15, 16]. Non-Gaussianity in the perturbation's mode could be relatively large in some situations (see for instance [17, 18, 19, 20, 21, 22, 23, 24, 25, 26]). It is now well-known that large non-Gaussianities naturally follow inflationary scenarios with higher derivative terms [15, 27]. One might well hope to detect this large non-Gaussianities by future improvements in experiments, which would provide another key observable to constraint or confirm particular inflation models and also the underlying high energy theories from which they are originated [15].

While higher derivative theories predict some levels of non-Gaussianities, these theories are typically plagued by ghost instabilities [28, 29]. In Horndeski construction these instabilities are avoided [30]. The most general Horndeski single-field Lagrangian leading to second-order equations of motion covers a wide variety of gravitational theories with one scalar degree of freedom. Among these theories one could mention standard slow-roll inflationary model [2, 3, 31, 32, 33, 34], non-minimally coupled scenarios [35, 36, 37], Brans-Dicke theories [38] (which include $f(R)$ gravity [39]), Galileon inflation [27, 40, 41, 42], k-inflation [43, 44] and field derivative coupled to gravity [45, 46, 47]). As a comprehensive study of the most general non-canonical and non-minimally coupled single field inflation models which yield second-order field equations, one can be referred to Kobayashi *et al.* [48]. In [48] the authors have presented the most general extension of the Galileons which are no longer based on a symmetry argument (since such a symmetry must be practically broken to terminate inflation and reheat the Universe). In order to give the stability analysis and the power spectrum of the primordial fluctuations in the generalized G-inflation, they have illustrated the generic behavior of the inflationary background and studied the nature of primordial perturbations at *linear* order, by obtaining the most general second-order action for both scalar and tensor density perturbations. On the other hand, these complex scenarios are capable to provide a dark energy component for late time dynamics of the universe [49, 50, 51, 52, 53, 54, 55, 56]. In these extended scenarios, the fields are considered to be non-minimally coupled to the background curvature leading to interesting cosmological outcomes in both dark energy and inflation eras. Since operators coming out from the non-minimal coupling are non-renormalizable, the unitarity bound of the theory during inflation era is violated

[57, 58, 59, 60]. One desires to avoid this unitarity violation to find a framework that the Higgs boson would behave like a primordial Inflaton. In this regard, considering non-minimal coupling between the derivatives of the scalar fields and curvature can be a solution [61, 62, 63, 64, 65, 66]. This scenario can be regarded as a subset of the most general scalar-tensor theories. In this respect, studying the second-order gravitational theories in connection with Galileon gravity [67, 68, 69] has attracted much attention recently. These models can be considered generally as rediscovery of Horndeski construction and its detailed features. To see how studies have been improved in this direction we refer to Refs. [39, 41, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90]. Galileon models with non-minimal derivative coupling have also frequently received attention as models of dark energy, because they can describe an effective short-distance theory associated with a modification of gravity on large scales [70, 71, 91, 92, 93].

In this paper, we study a particular Galileon inflation which is a radiatively stable higher derivative model of inflation. This model is determined by a finite number of relevant operators which are protected by a covariant generalization of the Galileon shift symmetry (see for instance [27, 94]). By comparing the obtained values of the observable parameters (such as the scalar spectral index and tensor-to-scalar ratio) in this setup with observation, viability of the model at hand can be tested to see whether it is successful or not. The combined WMAP9+eCMB+BAO+ H_0 data results in the constraints $r < 0.13$ and $n_s = 0.9636 \pm 0.0084$ [95], while the joint Planck2013+WMAP9+BAO data expresses the conditions $r < 0.12$ and $n_s = 0.9643 \pm 0.0059$ [96]. The more recent study of the Planck team has constrained the scalar spectral index and tensor-to-scalar ratio as $r < 0.099$ and $n_s = 0.9652 \pm 0.0047$, from Planck TT, TE, EE+low P+WP data [97, 98, 99]. We note that Planck TT, TE, EE+lowP refers to the combination of the likelihood at $l > 30$ using TT, TE, and EE spectra and also the low l multipole temperature polarization likelihood. Therefore, in order to compare our model with observational data, we study the behavior of the tensor-to-scalar ratio, r , versus the scalar spectral index, n_s , in the background of the Planck TT, TE, EE+low P data and find some constraints on the parameters space of the model. We focus our attention on several choices of functions of the scalar field in the action including $K(\phi)$, $\gamma(\phi)$ and the potential, $V(\phi)$. Furthermore, in each case, we analyze the deviation from the standard slow-roll consistency relation $r = -8n_T$ due to the effect of additional terms in the action. We note that the issue of consistency relation and its status in Galileon and DBI Galileon inflation are studied in [41, 42, 48, 100, 101]. In Ref. [100], the authors by considering restricted class of Galilean inflation models with Lagrangian $\mathcal{L}(\phi, X, \square\phi) = \frac{X^n \square\phi}{M^{4n-1}} - V(\phi)$ have shown that depending on the value of the parameter n , one can have both $r > -8n_T$ or $r < -8n_T$, in spite of the fact that the speed of sound is subluminal in both the cases. As we will show a slight deviation from the standard consistency relation can be obtained in our setup too.

We study also non-Gaussian feature of the perturbations numerically by focusing on the behavior of the orthogonal versus equilateral configuration in the background of the observational data. Non-Gaussianity will be detectable essentially by observation if the bispectrum, which is transformation of the three-point correlator in Fourier space, is of the order of the square of the power spectrum. Using different types of functions $K(\phi)$, $\gamma(\phi)$ and the potential $V(\phi)$, we show that for some ranges of the non-minimal coupling parameter, our model is consistent with observation and it is also possible to have large non-Gaussianity in this setup. In other words, we show that the non-Gaussianity of the primordial density perturbation generated during an epoch of Galileon inflation is a particularly powerful observational probe of these models. We also note that large non-Gaussianities would be observable by future improvements in experiments and in this respect this would be an important result in our study.

The paper is organized as follows: After describing the action for the Galileon inflationary model and deriving the main equations of the model in section II, we investigate the linear perturbation of the model in Section III. We expand the action up to the second order in perturbations and derive the two-point correlation function which gives the amplitude of the scalar perturbation and its spectral index. Furthermore, we study the tensor part of the perturbed metric and obtain the tensor perturbation and its spectral index. We also show that in the model under consideration, the consistency relation of the standard inflation is modified. In Sec. IV, we expand the action up to

the cubic order in perturbation to investigate nonlinear perturbation in the model. In this regard, the amplitude of the non-Gaussianity is derived in the equilateral and orthogonal configurations and with the equilateral limit, $k_1 = k_2 = k_3$. In this way, we focus on the possibility of deriving a large amount of non-Gaussianity in this particular Galileon inflation and obtain some conditions to have large non-Gaussianity. In Sec. V, we test our Galileon inflationary model in confrontation with the recently released observational data and find constraints on the parameters space of the model. We also test deviation from the standard consistency relation in each case. Finally, we give our summary and conclusions in Sec. VI.

2 The Model

The unique 4-dimensional action of the most general class of scalar-tensor theories which contains both first and second derivatives of the scalar field, and leads to the second order equations of motion for both the metric and scalar field, is given as follows

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{pl}^2}{2} R + \mathcal{L} \right), \quad (2.1)$$

where R is the Ricci scalar and represents the Einstein-Hilbert action of General Relativity and

$$\mathcal{L} \equiv \mathcal{L}(\phi, \nabla\phi, \nabla\nabla\phi) = \sum_{i=2}^5 \mathcal{L}_i, \quad (2.2)$$

which collects the higher order corrections to General Relativity. We should note that, in order to constitute a well-defined, predictive description of an inflationary phase, there are some conditions that the Lagrangian of this generated model, \mathcal{L} , must satisfy. An arbitrary action of the form (2.1) contains second and higher-order derivatives of the inflaton field, and therefore results in equations of motion of third-order or higher. Such theories do not usually possess a well-defined Cauchy problem [27, 102], (unless presenting an infinite number of derivatives), and this leads to appearance of ghost states which causes a pernicious loss of unitarity at the quantum level. For this reason, we should restrict our attention to special choices of \mathcal{L} which lead to second-order equations of motion [103, 104, 105]. In Ref. [106] it has been shown that choosing $\mathcal{L} = G(X, \phi)\Box\phi$ leads to second-order equations of motion for any choice of $G(X, \phi)$, and also unitary evolution as a quantum field theory. Thus, we consider such models as candidates for an inflationary action of the form (2.1) and define the following expressions for \mathcal{L}_i

$$\mathcal{L}_2 = K(\phi, X), \quad (2.3)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \quad (2.4)$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X} \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right], \quad (2.5)$$

$$\begin{aligned} \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu \nabla^\nu \phi) - \frac{1}{6}G_{5,X} \left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) + \right. \\ \left. 2(\nabla^\mu \nabla_\alpha \phi)(\nabla^\alpha \nabla_\beta \phi)(\nabla^\beta \nabla_\mu \phi) \right]. \end{aligned} \quad (2.6)$$

Here, ϕ is a scalar field coupled with gravity, $K(\phi, X)$ and $G_n(\phi, X)$ ($n = 3, 4, 5$) are ordinary functions in terms of the scalar field and $X = -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$, $R_{\mu\nu}$ is the Ricci tensor and $G_{\mu\nu}$ is the Einstein tensor ($G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$). $G_{,X}$ denotes derivative of G with respect to X . In this paper, we study how inflation can be realized in theories of the form (2.1).

Setting $\mathcal{L}_2 \neq 0$ and $G_n(\phi, X) = 0$ in the general action results the k-inflation model and in order to obtain the Lagrangian of G-inflation, we must set $\mathcal{L}_2, G_3 \neq 0$ and $G_4 = G_5 = 0$. However, by taking into account nonzero Lagrangians \mathcal{L}_4 and \mathcal{L}_5 , one can cover a wide variety of gravitational theories such as scalar-tensor theories, field derivative couplings with gravity, Galileon gravity and higher-curvature gravity including Gauss-Bonnet and f(R) theories.

In what follows we consider the case

$$K(X, \phi) = \mathcal{K}(\phi)X - V(\phi), \quad (2.7)$$

$$G_3(\phi, X) = -\gamma(\phi)X, \quad (2.8)$$

$$G_4(\phi, X) = \frac{1}{2} \left(\xi \phi^2 + \frac{X^2}{\mu^2} \right), \quad (2.9)$$

and

$$G_5(\phi, X) = 0, \quad (2.10)$$

where \mathcal{K} and γ are ordinary functions of the scalar field, V is the potential, μ is a constant and the first term in G_4 shows an explicit non-minimal coupling of the scalar field with the Ricci scalar with ξ being the non-minimal coupling parameter. Let us assume a spatially flat Friedmann-Robertson-Walker spacetime

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j. \quad (2.11)$$

Variation of the action with respect to the scalar field gives the following equation of motion

$$\begin{aligned} & \left(-4\gamma_{,\phi}X - \mathcal{K} + 6\gamma H\dot{\phi} - \frac{27H^2\dot{\phi}^2}{\mu^2} \right) \ddot{\phi} + \left(3\gamma\dot{\phi}^2 + 6\xi\phi - \frac{18H\dot{\phi}^3}{\mu^2} \right) \dot{H} - \\ & 3 \left(\frac{12H^2X}{\mu^2} + \mathcal{K} \right) H\dot{\phi} + \left(9H^2\gamma - \frac{1}{2}\mathcal{K}_{,\phi} - \gamma_{,\phi\phi}X \right) \dot{\phi}^2 + 12H^2\xi\phi - V_{,\phi} = 0, \end{aligned} \quad (2.12)$$

and varying the action with respect to the metric leads to the following Friedmann equations

$$3H^2 \left(M_{pl}^2 + \xi\phi^2 + \frac{X^2}{\mu^2} \right) - \left(\gamma_{,\phi}X + \frac{12H^2\dot{\phi}^2}{\mu^2} + \frac{1}{2}\mathcal{K} \right) \dot{\phi}^2 + 6\xi H\phi\dot{\phi} - V = 0, \quad (2.13)$$

and

$$\begin{aligned} -2\dot{H} \left(\frac{3X^2}{\mu^2} - \xi\phi^2 - M_{pl}^2 \right) &= 2\xi H\phi\dot{\phi} - \left(2\gamma_{,\phi}X + \frac{9H^2\dot{\phi}^2}{\mu^2} + \mathcal{K} + 2\xi \right) \dot{\phi}^2 - \\ & \left(2\xi\phi - \frac{6H\dot{\phi}^3}{\mu^2} + \gamma\dot{\phi}^2 \right) \ddot{\phi}, \end{aligned} \quad (2.14)$$

where a dot represents the derivative with respect to the cosmic time t . In this inflationary model, the slow-roll parameters which are defined as $\epsilon \equiv -\frac{\dot{H}}{H^2}$ and $\eta \equiv -\frac{1}{H}\frac{\ddot{H}}{\dot{H}}$ are given by the following expression

$$\epsilon = \frac{\mathcal{F}}{H^2 \left(M_{pl}^2 + \xi\phi^2 - 3\frac{X^2}{\mu^2} \right)}, \quad (2.15)$$

$$\eta = 2\epsilon - \frac{\dot{\mathcal{F}}}{H^3 \epsilon \left(M_{pl}^2 + \xi\phi^2 - 3\frac{X^2}{\mu^2} \right)} + \frac{2\mathcal{F}\dot{H}}{H^4 \epsilon \left(M_{pl}^2 + \xi\phi^2 - 3\frac{X^2}{\mu^2} \right)} - \frac{\mathcal{F} \left(6\frac{X}{\mu^2}\dot{\phi}\ddot{\phi} - 2\xi\phi\dot{\phi} \right)}{H^3 \epsilon \left(M_{pl}^2 + \xi\phi^2 - 3\frac{X^2}{\mu^2} \right)^2}, \quad (2.16)$$

where parameter \mathcal{F} is defined as

$$\mathcal{F} = \mathcal{K}X - \frac{12X^2}{\mu^2\dot{\phi}}\ddot{\phi} - 3H\gamma X\dot{\phi} + 2\gamma_{,\phi}X^2 + \frac{18H^2X^2}{\mu^2} - \xi H\phi\dot{\phi} + 2\xi X + \gamma X\ddot{\phi} + \xi\phi\ddot{\phi}. \quad (2.17)$$

Since during the inflationary era, the evolution of the Hubble parameter is so slow, the conditions $\epsilon \ll 1$ and $\eta \ll 1$ are satisfied in this regime and whenever one of these two conditions for the slow-varying parameters violates, the inflation phase terminates.

The minimal number of e-folds during inflationary regime is given by

$$N = \int_{t_*}^{t_f} H dt \quad (2.18)$$

and within the slow-roll limit, ($\ddot{\phi} \ll |3H\dot{\phi}|$ and $\dot{\phi}^2 \ll V(\phi)$), it takes the following form in our setup

$$N = \int_{\phi_*}^{\phi_f} \frac{\mathcal{K}V + 2\xi\phi V_{,\phi} - 2\xi^2 R\phi^2}{(M_{pl}^2 + \xi\phi^2)(\xi R\phi - V_{,\phi})} d\phi \quad (2.19)$$

Here, ϕ_* denotes the value of the inflaton field at the horizon crossing of the universe scale and ϕ_f determines its value at the time of exit from inflationary phase. Up to this point, we obtained the main equations of this extended inflationary model. In order to test this model, we proceed by studying the linear perturbation of the primordial fluctuations. To this end, we study the spectrum of perturbations which are produced by quantum fluctuations of the fields about their homogeneous background values.

3 Linear Perturbation

Now, we study linear perturbations of the model introduced in the previous section. These perturbations arise from the quantum behavior of both the space-time metric and the scalar field around the homogeneous background solutions.

At first, we should expand the action of the model up to the second order of small fluctuations. In this regard, it is convenient to use the Arnowitt-Deser-Misner (ADM) formalism in which, by choosing a suitable gauge, one can eliminate one extra degree of freedom of perturbations from the beginning of the calculation [107]. The space-time metric in this formalism is given by

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (3.1)$$

where N and N^i are the lapse function and the shift vector, respectively. One can obtain the general perturbed form of the metric by expanding the lapse function as $N = 1 + \mathcal{B}$ and the shift vector as $N^i = \delta^{ij}\partial_j\zeta + v^i$ in which \mathcal{B} and ζ are 3-scalar and v^i is a vector satisfying the condition $v^i_{,i} = 0$ [108, 109]. We note that, since the second order perturbation is multiplied by a factor which is vanishing using the first order solution, it is sufficient to compute N or N^i up to the first order. The contribution of the third order term also vanishes because of multiplying by a constraint equation at the zeroth order obeying the equations of motion [13, 29, 110]. The coefficient h_{ij} should be written as $h_{ij} = a^2[(1 - 2\Theta)\delta_{ij} + 2\mathcal{T}_{ij}]$, with Θ being the spatial curvature perturbation and \mathcal{T}_{ij} being a spatial shear 3-tensor which is both symmetric and traceless. Finally, the above perturbed metric (3.1) takes the form

$$ds^2 = -(1 + 2\mathcal{B})dt^2 + 2a(t)\zeta_{,i}dtdx^i + a^2(t)\left[(1 - 2\Theta)\delta_{ij} + 2\mathcal{T}_{ij}\right]dx^i dx^j. \quad (3.2)$$

Now, in order to study the scalar perturbation of the theory, one can choose the uniform-field gauge in which $\delta\phi = 0$, which fixes the time-component of the gauge-transformation vector ξ^μ , and also the gauge $\mathcal{T}_{ij} = 0$. Considering the scalar part of the perturbations at the linear level and within the uniform-field gauge, finally the perturbed metric can be rewritten as [108, 109, 111]

$$ds^2 = -(1 + 2\mathcal{B})dt^2 + 2a(t)\zeta_{,i}dx^i dt + a^2(t)(1 - 2\Theta)\delta_{ij}dx^i dx^j. \quad (3.3)$$

Upon integrating out the auxiliary fields \mathcal{B} and ζ (see Appendix **A**), one obtains for the quadratic action

$$S_2 = \int dt d^3x a^3 \mathcal{X} \left[\dot{\Theta}^2 - \frac{c_s^2}{a^2} (\partial\Theta)^2 \right], \quad (3.4)$$

where the parameters \mathcal{X} and c_s^2 (known as the sound speed squared), are expressed as

$$\mathcal{X} = \frac{4(M_{pl}^2 + 2G_4 - 4XG_{4,X})^2 q_2}{3q_1^2} + 3(M_{pl}^2 + 2G_4 - 4XG_{4,X}), \quad (3.5)$$

and

$$\begin{aligned} c_s^2 = & 3 \left[2H(M_{pl}^2 + 2G_4 - 4XG_{4,X})^2 q_1 - (M_{pl}^2 + 2G_4) q_1^2 + \right. \\ & 4(M_{pl}^2 + 2G_4 - 4XG_{4,X}) \frac{d}{dt} (M_{pl}^2 + 2G_4 - 4XG_{4,X}) q_1 - 2(M_{pl}^2 + 2G_4 - 4XG_{4,X})^2 \frac{d}{dt} q_1 \left. \right] \times \\ & [(M_{pl}^2 + 2G_4 - 4XG_{4,X})(4(M_{pl}^2 + 2G_4 - 4XG_{4,X}) q_2 + 9q_1^2)]^{-1}, \end{aligned} \quad (3.6)$$

where q_1 and q_2 are given by

$$q_1 = 2H(M_{pl}^2 + 2G_4) - 2X\dot{\phi}G_{3,X} - 16H(XG_{4,X} + X^2G_{4,XX}) + 2\dot{\phi}(G_{4,\phi} + 2XG_{4,\phi X}), \quad (3.7)$$

and

$$\begin{aligned} q_2 = & -9H^2(M_{pl}^2 + 2G_4) + 3(XK_{,X} + 2X^2K_{,XX}) + 18H\dot{\phi}(2XG_{3,X} + X^2G_{3,XX}) - \\ & 6X(G_{3,\phi} + XG_{3,\phi X}) + 18H^2(7XG_{4,X} + 16X^2G_{4,XX} + 4X^3G_{4,XXX}) - \\ & 18H\dot{\phi}(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2G_{4,\phi XX}). \end{aligned} \quad (3.8)$$

More details in deriving these type of equations can be found in Refs. [16, 20, 29, 88].

Let us now proceed with the spatial curvature perturbation, Θ . Varying the action (3.4) yields the following equation of motion

$$\ddot{\Theta} + \left(3H + \frac{\dot{\mathcal{X}}}{\mathcal{X}} \right) \dot{\Theta} + c_s^2 \frac{k^2}{a^2} \Theta = 0. \quad (3.9)$$

Up to the lowest order of the slow-roll approximation, the solution of this equation is given by the following expression

$$\Theta = \frac{iH e^{-ic_s^2 k \tau}}{2(c_s k)^{3/2} \sqrt{\mathcal{X}}} (1 + ic_s k \tau). \quad (3.10)$$

In order to study the power spectrum of the curvature perturbation for the model at hand, one needs to obtain the two-point correlation function. The two-point correlation function of curvature perturbations can be derived by obtaining the vacuum expectation value of Θ at $\tau = 0$ (which corresponds to the end of the inflation phase), as

$$\langle 0 | \Theta(0, \mathbf{k}_1) \Theta(0, \mathbf{k}_2) | 0 \rangle = \frac{2\pi^2}{k^3} (2\pi)^3 \mathcal{A}_s \delta^3(\mathbf{k}_1 + \mathbf{k}_2). \quad (3.11)$$

Here, \mathcal{A}_s is called the power spectrum of the scalar perturbations which is defined as

$$\mathcal{A}_s = \frac{H^2}{8\pi^2 \mathcal{X} c_s^3}. \quad (3.12)$$

The scalar spectral index of the perturbations at $c_s k = aH$ (which corresponds to the time of Hubble crossing, with k being the wave number) is defined as follows

$$n_s - 1 = \frac{d \ln \mathcal{A}_s}{d \ln k}. \quad (3.13)$$

Before obtaining the scalar spectral index in our model, it is convenient to introduce the following parameter

$$\mathcal{E}_s = \frac{c_s^2 \mathcal{X}}{M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2}}, \quad (3.14)$$

which using (3.5) and (3.6), we can rewrite it as

$$\mathcal{E}_s = \epsilon + \frac{(\xi \phi - \gamma X) \dot{\phi} + 8 \frac{X^2}{\mu^2}}{H \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right)} + \mathcal{O}(\epsilon^2). \quad (3.15)$$

Then equation (3.12) can be rewritten as follows

$$\mathcal{A}_s = \frac{H^2}{8\pi^2 \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right) \mathcal{E}_s c_s}. \quad (3.16)$$

Finally the scalar spectral index in our model can be derived as follows

$$n_s - 1 = -2\epsilon - \frac{1}{H} \frac{\dot{c}_s}{c_s} - \frac{1}{H} \frac{\dot{\mathcal{E}}_s}{\mathcal{E}_s} - \frac{1}{H} \frac{d}{dt} \ln \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right). \quad (3.17)$$

Any deviation of n_s from the unity confirms the scale dependance of the perturbations.

Let us now proceed further by studying the amplitude of the tensor perturbation and its spectral index. In order to obtain the power spectrum of the gravitational waves in this setup, we study the tensor perturbations of the form

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij}^{TT})dx^i dx^j, \quad (3.18)$$

which is the tensor part of the perturbed metric (3.2). h_{ij}^{TT} can be decomposed in terms of the two polarization modes as follows

$$h_{ij}^{TT} = h_+ e_{ij}^+ + h_\times e_{ij}^\times \quad (3.19)$$

where $e_{ij}^{(\times,+)}$ are symmetric, traceless and transverse tensors. In this case, the second-order action for the tensor mode can be written as follows

$$S_T = \int dt d^3x a^3 \mathcal{X}_T \left[\dot{h}_{(+)}^2 - \frac{c_T^2}{a^2} (\partial h_{(+)}^2) + \dot{h}_{(\times)}^2 - \frac{c_T^2}{a^2} (\partial h_{(\times)}^2) \right], \quad (3.20)$$

where the parameters \mathcal{X}_T and c_T^2 are defined as

$$\mathcal{X}_T = \frac{1}{4} \left(M_{pl}^2 + \xi \phi^2 - 3 \frac{X^2}{\mu^2} \right), \quad (3.21)$$

and

$$c_T^2 = \frac{M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2}}{M_{pl}^2 + \xi \phi^2 - 3 \frac{X^2}{\mu^2}}, \quad (3.22)$$

respectively. Following the strategy performed for the scalar perturbations, the amplitude of the tensor perturbations is obtained as

$$\mathcal{A}_T = \frac{H^2}{2\pi^2 \mathcal{X}_T c_T^3}. \quad (3.23)$$

Using the definition of the spectral index of the gravitational waves

$$n_T = \frac{d \ln \mathcal{A}_T}{d \ln k}, \quad (3.24)$$

we obtain the following expression for n_T

$$n_T = -2\epsilon - \frac{2}{\left(M_{pl}^2 + \xi\phi^2 + \frac{X^2}{\mu^2}\right)} \left[\frac{\xi\phi\dot{\phi}}{H} + \frac{2X^2\ddot{\phi}}{\mu^2 H\dot{\phi}} \right], \quad (3.25)$$

which using (3.15) can be rewritten as

$$n_T = -2\mathcal{E}_s - \frac{2\gamma X\dot{\phi} - 16H\frac{X^2}{\mu^2}}{H\left(M_{pl}^2 + \xi\phi^2 + \frac{X^2}{\mu^2}\right)} + \mathcal{O}(\epsilon^2), \quad (3.26)$$

equivalently. The tensor-to-scalar ratio is another important inflationary parameter which provides some information about the perturbation and is defined as

$$r = \frac{\mathcal{A}_T}{\mathcal{A}_s}. \quad (3.27)$$

This parameter takes the following form in our setup,

$$r = 16 \frac{c_s}{c_T} \mathcal{E}_s. \quad (3.28)$$

Using equation (3.26) in (3.28), leads to the following expression for r

$$r \simeq -8c_s \left[n_T + \frac{2\gamma X\dot{\phi} - 16H\frac{X^2}{\mu^2}}{H\left(M_{pl}^2 + \xi\phi^2 + \frac{X^2}{\mu^2}\right)} \right]. \quad (3.29)$$

As we know, there is a relation between r and n_s in the standard inflationary model as $r = -8c_s n_T$ which is known as the consistency relation. Equation (3.29) shows that in our model the consistency relation of the standard inflation is modified, which is because of the presence of the terms G_3 and G_4 in our action. Up to now, we obtained the primordial fluctuations in linear order. In the next section, we explore the non-Gaussianity of the density perturbations using the nonlinear perturbations.

4 Nonlinear Perturbations and Non-Gaussianity

In this section, we study the non-Gaussianity of the primordial density perturbation which is another important aspect of an inflationary model. In order to compute the amount of non-Gaussianity in a specific inflation model, we should go beyond the linear order perturbation theory. In other words, the two-point correlation function of the scalar perturbations gives no information about the non-Gaussian distribution of the primordial perturbations in the model. Thus, one has to study higher order correlation functions. Since for a Gaussian perturbation, all odd n -point correlators vanish and the higher even n -point correlation functions are expressed in terms of sum of products of the two-point functions, so, we should study the three-point correlation function in order to explore the non-Gaussianity of the density perturbations. To this end, we should expand the action of the model up to the cubic order in the small fluctuations around the homogeneous background solution.

We should remind that cubic terms obtained in this manner lead to a change both in the ground state of the quantum field and nonlinearities in the evolution. After expanding the action (1) up to the third order in perturbation, one should eliminate the perturbation parameter \mathcal{B} in the expanded action. By introducing an auxiliary field \mathcal{Q} which satisfies the following expressions

$$\zeta = \left[\frac{a^2}{M_{pl}^2 + \xi\phi^2 - 3\frac{X^2}{\mu^2}} \right] \mathcal{Q} + \left[\frac{M_{pl}^2 + \xi\phi^2 - 3\frac{X^2}{\mu^2}}{H \left(M_{pl}^2 + \xi\phi^2 - 15\frac{X^2}{\mu^2} \right) + \gamma X \dot{\phi} + \xi \phi \dot{\phi}} \right] \Theta, \quad (4.1)$$

and

$$\partial^2 \mathcal{Q} = \mathcal{X} \dot{\Theta}, \quad (4.2)$$

one can obtain the third order action up to the leading order (see Appendix B). After obtaining the third order action (6.4), we are in the position to study the non-Gaussianity of the primordial perturbations by evaluating the three-point correlation functions. To this end, we use the interacting picture and obtain the vacuum expectation value of the curvature perturbation Θ for the three-point operator in the conformal time interval between the beginning of the inflation, τ_i , and the end of the inflation, τ_f as follows (see for instance [13, 16, 88])

$$\langle \Theta(\mathbf{k}_1) \Theta(\mathbf{k}_2) \Theta(\mathbf{k}_3) \rangle = -i \int_{\tau_i}^{\tau_f} d\tau a \langle 0 | [\Theta(\tau_f, \mathbf{k}_1) \Theta(\tau_f, \mathbf{k}_2) \Theta(\tau_f, \mathbf{k}_3), \mathcal{H}_{int}(\tau)] | 0 \rangle. \quad (4.3)$$

with \mathcal{H}_{int} being the interacting Hamiltonian which is equal to the Lagrangian of the cubic action. Since the coefficients in the brackets of the Lagrangian (6.4) vary slower than the scale factor, we can approximate these coefficients to be constants and solve the integral of equation (4.3), which leads to the following three-point correlation function of the curvature perturbation in the Fourier space

$$\langle \Theta(\mathbf{k}_1) \Theta(\mathbf{k}_2) \Theta(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) A_s^2 \Xi_{\Theta}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3), \quad (4.4)$$

with A_s being the power spectrum of perturbation some time after the Hubble radius crossing (given by equation (3.16)), and Ξ being defined as

$$\Xi_{\Theta}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{(2\pi)^4}{\prod_{i=1}^3 k_i^3} \mathcal{G}_{\Theta}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3). \quad (4.5)$$

Also, the parameter \mathcal{G}_{Θ} is defined as

$$\mathcal{G}_{\Theta} = \frac{3}{4} \left(1 - \frac{1}{c_s^2} \right) \mathcal{Z}_1 + \frac{1}{4} \left(1 - \frac{1}{c_s^2} \right) \mathcal{Z}_2 + \frac{3}{2} \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma} + \Gamma \right) \mathcal{Z}_3. \quad (4.6)$$

in which \mathcal{Z}_1 , \mathcal{Z}_2 and \mathcal{Z}_3 are the shape functions with the following relations

$$\mathcal{Z}_1 = \frac{2}{K} \sum_{i>j} k_i^2 k_j^2 - \frac{1}{K^2} \sum_{i \neq j} k_i^2 k_j^3, \quad (4.7)$$

$$\mathcal{Z}_2 = \frac{1}{2} \sum_i k_i^3 + \frac{2}{K} \sum_{i>j} k_i^2 k_j^2 - \frac{1}{K^2} \sum_{i \neq j} k_i^2 k_j^3, \quad (4.8)$$

$$\mathcal{Z}_3 = \frac{(k_1 k_2 k_3)^2}{K^3}, \quad (4.9)$$

where

$$K = \sum_i k_i. \quad (4.10)$$

Equation (4.5) shows that the three-point correlator depends on the three momenta k_1 , k_2 and k_3 . We note that, in order to satisfy the translation invariance, these momenta should form a closed triangle with the constraint $k_1 + k_2 + k_3 = 0$ [15, 86, 112, 113, 114]. Furthermore, considering the rotational invariance makes the shape of the triangle important. Different amount of momenta, gives different shapes of triangle and each shape has a maximal signal in a special configuration. The simplest one is the local shape [115, 116, 117, 118], having a peak in the squeezed limit with $k_3 \rightarrow 0$ and $k_1 \simeq k_2$. There is another shape which corresponds to the equilateral configuration [15] with a signal at $k_1 = k_2 = k_3$. A shape of non-Gaussianity whose scalar product with the equilateral shape vanishes, is called the orthogonal configuration [119]. A linear combination of the equilateral and orthogonal configurations results in a shape corresponding to folded triangle [120] which is orthogonal to the equilateral templates and has a maximal signal at the $k_1 = 2k_2 = 2k_3$ limit. We also mention that, the orthogonal configuration has a signal with a positive peak at the equilateral configuration and a negative peak at the folded configuration. From the bispectrum \mathcal{G}_Θ of the three-point correlation function of curvature perturbations, the dimensionless parameter, f_{NL} , characterizing the amplitude of non-Gaussianities, which is called nonlinearity parameter, is defined as

$$f_{NL} = \frac{10}{3} \frac{\mathcal{G}_\Theta}{\sum_{i=1}^3 k_i^3}. \quad (4.11)$$

As has been noted previously, purely adiabatic Gaussian perturbations result $f_{NL} = 0$, however, the presence of non-Gaussian perturbations leads to deviation from $f_{NL} = 0$. In what follows, we investigate the amplitude of non-Gaussianity in the equilateral and orthogonal configurations. In this regard, the bispectrum \mathcal{G}_Θ in these configurations should be obtained. To this end, we follow [121, 122, 123] and by considering $\mathcal{G}_\Theta = \sum_{i=1}^3 \mathcal{G}_\Theta^i$, we introduce the quantity \mathcal{I} by the following expression

$$\mathcal{I}(\Xi_\Theta^{(i)}, \Xi_\Theta^{(j)}) = \frac{\mathcal{U}(\Xi_\Theta^{(i)}, \Xi_\Theta^{(j)})}{\sqrt{\mathcal{U}(\Xi_\Theta^{(i)}, \Xi_\Theta^{(i)}) \mathcal{U}(\Xi_\Theta^{(j)}, \Xi_\Theta^{(j)})}}, \quad (4.12)$$

where

$$\mathcal{U}(\Xi_\Theta^{(i)}, \Xi_\Theta^{(j)}) = \int dk_1 dk_2 dk_3 \Xi_\Theta^{(i)}(k_1, k_2, k_3) \Xi_\Theta^{(j)}(k_1, k_2, k_3) \frac{(k_1 k_2 k_3)^4}{(k_1 + k_2 + k_3)^3}. \quad (4.13)$$

This integration should be done in the following region

$$0 \leq k_1 < \infty, \quad 0 < \frac{k_2}{k_1} < 1, \quad 1 - \frac{k_2}{k_1} \leq \frac{k_3}{k_1} \leq 1. \quad (4.14)$$

For $|\mathcal{I}(\Xi_\Theta^{(i)}, \Xi_\Theta^{(j)})| \simeq 1$ the correlation is large, whereas for $|\mathcal{I}(\Xi_\Theta^{(i)}, \Xi_\Theta^{(j)})| \simeq 0$ the two shapes are almost orthogonal with a small correlation. Following [122] we define a shape \mathcal{Z}_*^{equil} as

$$\mathcal{Z}_*^{equil} = -\frac{12}{13} (3\mathcal{Z}_1 - \mathcal{Z}_2). \quad (4.15)$$

Furthermore, we introduce another shape which is exactly orthogonal to \mathcal{Z}_*^{equil} , as

$$\mathcal{Z}_*^{ortho} = \frac{12}{14 - 13\beta} [\beta(3\mathcal{Z}_1 - \mathcal{Z}_2) + 3\mathcal{Z}_1 - \mathcal{Z}_2], \quad (4.16)$$

where $\beta = 1.1967996$. Finally, using these relations, the leading-order bispectrum (4.6) can be written in terms of the equilateral and orthogonal basis, \mathcal{Z}_*^{equil} and \mathcal{Z}_*^{ortho} , as the following expressions

$$\mathcal{G}_\Theta = \mathcal{C}_1 \mathcal{Z}_*^{equil} + \mathcal{C}_2 \mathcal{Z}_*^{ortho}, \quad (4.17)$$

with C_1 and C_2 being coefficients which determine the magnitudes of the three-point correlation function arising from equilateral and orthogonal contributions, respectively, and are expressed as

$$C_1 = \frac{13}{12} \left[\frac{1}{24} \left(1 - \frac{1}{c_s^2} \right) (2 + 3\beta) + \frac{\lambda}{12\Sigma} (2 - 3\beta) + \left(\frac{1}{3\mathcal{E}_s H \left(M_{pl}^2 + \xi\phi^2 + \frac{X^2}{\mu^2} \right)} \right) \times \right. \quad (4.18)$$

$$\left. \left(\gamma X \dot{\phi} \left(\frac{2 - 3\beta}{2} - \frac{1}{c_s^2} \right) - 6H \frac{X^2}{\mu^2} \left(2 - 3\beta - \frac{1}{c_s^2} \right) \right) \right],$$

and

$$C_2 = \frac{14 - 13\beta}{12} \left[\frac{1}{8} \left(1 - \frac{1}{c_s^2} \right) - \frac{\lambda}{4\Sigma} + \frac{12H \frac{X^2}{\mu^2} - \gamma X \dot{\phi}}{2\mathcal{E}_s H \left(M_{pl}^2 + \xi\phi^2 + \frac{X^2}{\mu^2} \right)} \right]. \quad (4.19)$$

Here, λ and Σ are defined by equations (6.5) and (6.6), respectively. By using equations (4.12)-(4.19), and also by definition of the non-linearity parameter (4.11), we find the following expressions for the amplitude of non-Gaussianity in the equilateral and orthogonal configurations respectively

$$f_{NL}^{equil} = \left(\frac{130}{36 \sum_{i=1}^3 k_i^3} \right) \left[\frac{1}{24} \left(1 - \frac{1}{c_s^2} \right) (2 + 3\beta) + \frac{\lambda}{12\Sigma} (2 - 3\beta) + \right. \quad (4.20)$$

$$\left. \left(\gamma X \dot{\phi} \left(\frac{2 - 3\beta}{2} - \frac{1}{c_s^2} \right) - \left(6H \frac{X^2}{\mu^2} \right) \left(2 - 3\beta - \frac{1}{c_s^2} \right) \right) \left(\frac{1}{3\mathcal{E}_s H \left(M_{pl}^2 + \xi\phi^2 + \frac{X^2}{\mu^2} \right)} \right) \right] \mathcal{Z}_*^{equil},$$

and

$$f_{NL}^{ortho} = \left(\frac{140 - 130\beta}{36 \sum_{i=1}^3 k_i^3} \right) \left[\frac{1}{8} \left(1 - \frac{1}{c_s^2} \right) - \frac{\lambda}{4\Sigma} + \frac{12H \frac{X^2}{\mu^2} - \gamma X \dot{\phi}}{2\mathcal{E}_s H \left(M_{pl}^2 + \xi\phi^2 + \frac{X^2}{\mu^2} \right)} \right] \mathcal{Z}_*^{ortho}. \quad (4.21)$$

We again emphasize that, the shape function in the equilateral configuration has a peak at the equilateral limit, ($k_1 = k_2 = k_3$). Moreover, the orthogonal shape has a signal with a positive peak at the equilateral configuration. Therefore, the nonlinearity parameter in both configurations can be rewritten as

$$f_{NL}^{equil} = \frac{325}{18} \left[\frac{1}{24} \left(1 - \frac{1}{c_s^2} \right) (2 + 3\beta) + \frac{\lambda}{12\Sigma} (2 - 3\beta) + \left(\gamma X \dot{\phi} \left(\frac{2 - 3\beta}{2} - \frac{1}{c_s^2} \right) - \right. \quad (4.22)$$

$$\left. \left(6H \frac{X^2}{\mu^2} \right) \left(2 - 3\beta - \frac{1}{c_s^2} \right) \right) \left(\frac{1}{3\mathcal{E}_s H \left(M_{pl}^2 + \xi\phi^2 + \frac{X^2}{\mu^2} \right)} \right) \right],$$

and

$$f_{NL}^{ortho} = \frac{10}{9} \left(\frac{65}{4} \beta + \frac{7}{6} \right) \left[\frac{1}{8} \left(1 - \frac{1}{c_s^2} \right) - \frac{\lambda}{4\Sigma} + \frac{12H \frac{X^2}{\mu^2} - \gamma X \dot{\phi}}{2\mathcal{E}_s H \left(M_{pl}^2 + \xi\phi^2 + \frac{X^2}{\mu^2} \right)} \right]. \quad (4.23)$$

As Burrage et al. [27] have mentioned, the non-Gaussianity is not constrained to obey $f_{NL} \propto \frac{1}{c_s^2}$ in Galileon inflation as a new class of higher derivative inflationary models. We see that this is actually the case in our setup too. Non-Gaussianities are not just in the form of $f_{NL} \propto \frac{1}{c_s^2}$. As equations (4.22) and (4.23) show, in the presence of higher order derivatives of the Galileon field, non-gaussianities have more complicated structure than the simple $f_{NL} \propto \frac{1}{c_s^2}$ relation.

Up to this point, we have obtained the main equations of the model at hand. In the next section, we test our inflationary model in confrontation with Planck 2015 TT, TE, EE + low P and Planck2015 TTT, EEE, TTE and EET joint data to see the consistency and viability of this model. We also find some constraints on the model's parameters space (especially the non-minimal coupling parameter) in this treatment.

5 Observational Constraints

In previous sections the primordial fluctuations in both linear and nonlinear orders have been obtained. However, the viability of an inflationary model depends on its perturbation parameters coincidence with observational data. Thus, in what follows we find some observational constraints on the parameters space of the Galileon inflation treated in this paper. In this regard, we should firstly define the form of the functions of the scalar field introduced in action (2.1), $V(\phi)$, $\mathcal{K}(\phi)$ and $\gamma(\phi)$. We adopt these functions as $V(\phi) \sim \phi^m$, $\mathcal{K}(\phi) \sim \phi^m$ and $\gamma(\phi) \sim \phi^m$ ($m = 1$ corresponds to a linear potential, $m = 2$ refers to a quadratic potential and $m = 3$ and $m = 4$ imply cubic and quartic potentials, respectively). After choosing these forms of the generic functions, we analyze our model numerically and find some constraints on the parameters space of the model. For this purpose, at first, by substituting these functions into the integral of equation (2.19) and solving this integral, we obtain the value of the inflaton field at the horizon crossing of the physical scales in terms of the minimal number of e-folds, N . Next, we use the obtained value of the field in equations (3.17), (3.25), (3.29), (4.22) and (4.23) in order to calculate the spectral index of both scalar and tensor perturbations modes, tensor-to-scalar ratio and the amplitudes of the equilateral and orthogonal configurations of the non-Gaussianity in terms of the minimal number of e-folds. Now, we can explore the cosmological parameters numerically to see the observational viability of this setup in confrontation with recently released observational data. Another aspect of our study is the deviation from the standard consistency relation of the single-field inflation, $r = -8n_T$. Since Galileon inflation is a generalized theory, we detect deviations away from the standard consistency relation.

Here we perform our analysis by setting $m = 1, 2, 3, 4$, (taking four types of potentials), and $N = 50, 60$ and 70 for each values of m . The results are shown in figures 1, 2, 3, 4, 5 and 6. These figures confirm that a Galileon inflationary model as described by the action (2.1), with nonzero G_3 and G_4 (which indicates the existence of a non-minimal coupling between the scalar field ϕ and the Ricci scalar R), in some ranges of the non-minimal coupling parameter, ξ , is consistent with Planck2015 data. The ranges of the acceptable values of the non-minimal coupling are shown in tables 1 and 2.

Fig. 1 shows the behavior of the tensor-to-scalar ratio versus the scalar spectral index (left panel) with $m = 1$ (which corresponds to $V(\phi) \sim \phi$, $\mathcal{K} \sim \phi$ and $\gamma \sim \phi$) for $N = 50, 60$ and 70 in the background of the Planck2015 TT, TE, EE+lowP data. Our analysis shows that choosing a linear form of the field for the functions V , \mathcal{K} and γ , a Galileon inflation is in a good agreement with recent data if $0.0756845 < \xi < 0.0756980$ for $N = 50$, $0.0679086 < \xi < 0.067919$ for $N = 60$ and $0.061955 < \xi < 0.0619636$ for $N = 70$. However, the right panel of this figure shows the ratio $\frac{r}{n_T}$ of the model as a function of the sound speed c_s , which shows deviation from the standard consistency relation, $r = -8n_T$, for $m = 1$.

In Fig. 2 the behavior of r versus n_s for $m = 2$ is plotted (left panel) which shows that adopting a quadratic form of the functions V , \mathcal{K} and γ is a suitable choice for our Galileon inflation. Furthermore, as an important result, we find a slight deviation from the standard consistency relation in this case. Nevertheless, such a deviation seems not to be enough significant to be detected with confidence.

In Fig. 3 we depicted our results for $m = 3$ which is consistent with observation for some ranges of ξ that is given in Table 1. The results of our analysis for the quartic functions of the scalar field are drawn in Fig. 4. This figure shows that choosing $m = 4$, the Galileon inflation is consistent with recent data for $N = 60$ and $N = 70$. Deviation from the standard slow-roll consistency relation is shown in the right panel of each figures. One can see the ranges of the non-minimal coupling parameter, ξ , in which the values of the inflationary parameters r and n_s are compatible with the 95% CL of the Planck2015 TT, TE, EE+ low P joint data, in Table 1.

The numerical analysis on the non-Gaussian feature of the perturbations in a Galileon inflation has been performed too. The results are shown in Figs. 5 and 6. As another important result in our treatment, in some ranges of the NMC parameter, it is possible to have large non-Gaussianity. For instance, for $N = 60$ and $m = 2$, large non-Gaussianity can be realized for region $\xi < 0.00963$ in this setup. The ranges of the non-minimal coupling parameter, ξ , in which the values of the inflationary parameters f_{NL}^{ortho} and f_{NL}^{equi} are compatible with the 95% CL of the Planck2015 TTT, EEE, TTE

Table 1. The ranges of the non-minimal coupling parameter, ξ , in which the values of the inflationary parameters r and n_s are compatible with the 95% CL of the Planck2015 TT, TE, EE+low P joint data.

	$N = 50$	$N = 60$	$N = 70$
$m = 1$	$0.0756845 < \xi < 0.0756980$	$0.0679086 < \xi < 0.067919$	$0.061955 < \xi < 0.0619636$
$m = 2$	$0.007295 < \xi < 0.00769$	$0.00675 < \xi < 0.007115$	$0.006325 < \xi < 0.00667$
$m = 3$	$0.009302 < \xi < 0.009535$	$0.009162 < \xi < 0.009348$	$0.009054 < \xi < 0.009205$
$m = 4$	<i>not consistant</i>	$0.02345 < \xi < 0.0249$	$0.0236 < \xi < 0.0285$

Table 2. The ranges of the non-minimal coupling parameter, ξ , in which the values of the inflationary parameters f_{NL}^{ortho} and f_{NL}^{equi} are compatible with the 95% CL of the Planck2015 TTT, EEE, TTE and EET joint data.

	$N = 50$	$N = 60$	$N = 70$
$m = 1$	$0.06152 < \xi < 0.07619$	$0.06142 < \xi < 0.08345$	$0.0603 < \xi < 0.08486$
$m = 2$	$\xi < 0.0102$	$\xi < 0.00963$	$\xi < 0.009384$
$m = 3$	$\xi < 0.01563$	$\xi < 0.01553$	$\xi < 0.01545$
$m = 4$	$0.01697 < \xi < 0.02872$	$0.01723 < \xi < 0.0308$	$0.01785 < \xi < 0.03412$

and EET joint data are given in Table 2.

6 Summary and Conclusions

Successful inflationary models must obey an approximate shift symmetry to derive enough e-folds of inflation. Using this shift symmetry we are allowed to add any scalar constructed from gradients of the field to the inflationary Lagrangian. However, we must be careful that adding arbitrary higher derivative operators to the inflationary Lagrangian can lead to a loss of unitarity, because of the appearance of ghost states [67]. While one has not to be worry about protection of the resulting Lagrangian from large renormalizations, adding more input parameters than those can be measured destroys the predictivity of the theory [27].

With these points in mind, in this paper we have studied a Galileon inflation, which avoids the mentioned difficulties. We have considered an inflationary Lagrangian containing non-canonical derivative operators. The form of these operators are protected by the covariant generalization of the Galileon shift symmetry. This scenario contains a finite number of operators which lead to second order field equations, implying the absence of ghosts.

We have firstly found main equations of the inflationary dynamics in a Galileon inflation and then using the ADM formulation of the metric, we have studied the linear perturbations in this setup.

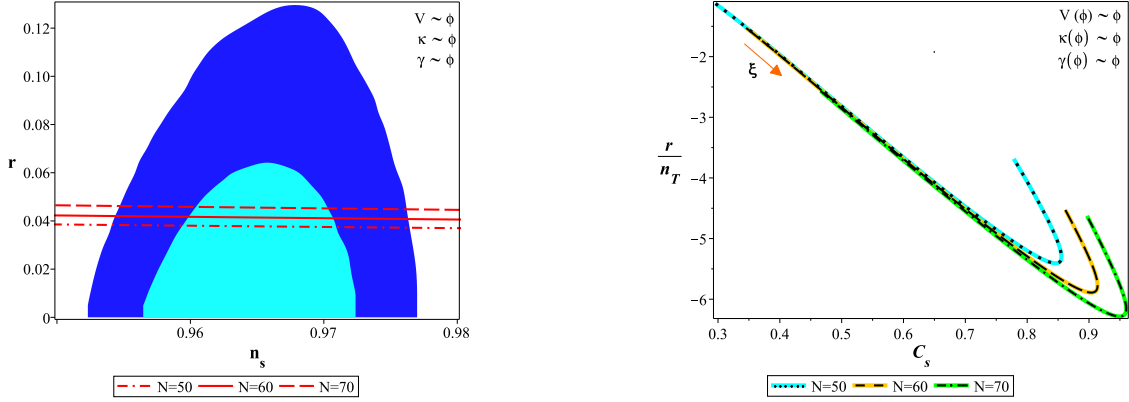


Figure 1. In the left panel the tensor-to-scalar ratio versus the scalar spectral index in the background of the Planck2015 TT, TE, EE+lowP data for a Galileon inflationary model is plotted with $m = 1$, which corresponds to potential of the form $V \sim \phi$ and also $\mathcal{K} \sim \phi$ and $\gamma \sim \phi$ for $N = 50, 60$ and 70 . Using the constraints obtained for ξ in this case, the ratio $\frac{r}{n_T}$ for the model is plotted as a function of the sound speed, c_s , for each e-folds number in the right panel. This figure shows deviation from the standard consistency relation $r = -8n_T$.

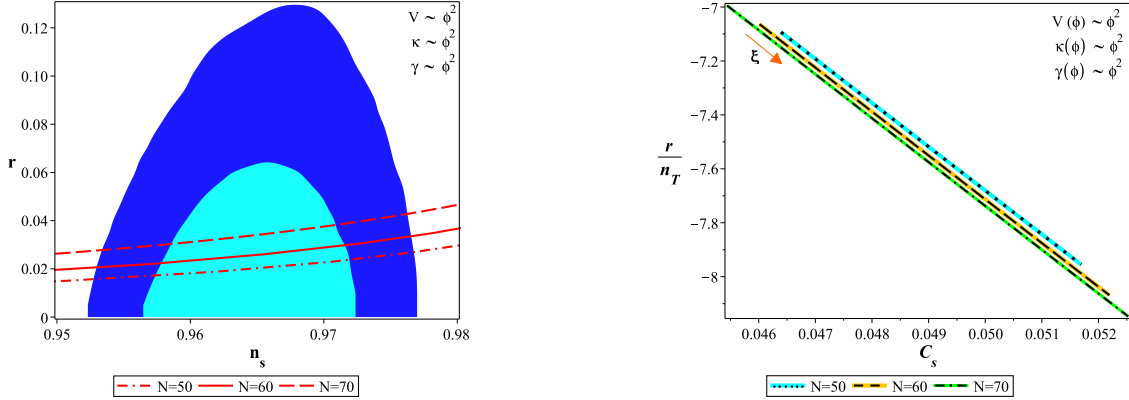


Figure 2. In the left panel the tensor-to-scalar ratio versus the scalar spectral index in the background of the Planck2015 TT, TE, EE+lowP data for a Galileon inflationary model is plotted with $m = 2$, which corresponds to potential of the form $V \sim \phi^2$ and also $\mathcal{K} \sim \phi^2$ and $\gamma \sim \phi^2$ for $N = 50, 60$ and 70 . Using the constraints obtained for ξ in this case, the ratio $\frac{r}{n_T}$ for the model is plotted as a function of the sound speed, c_s , for each e-folds number in the right panel. We find a *slight* deviation from the standard consistency relation $r = -8n_T$ which seems not to be enough significant to be detected with confidence.

By expanding the action up to the second order in perturbations in our model, we have derived the two-point correlation functions and obtained the amplitude of the scalar perturbation and its spectral index. We have also derived the tensor perturbation of the model and its spectral index by studying the tensor part of the perturbed metric. The ratio between the amplitude of the tensor and scalar perturbations (tensor-to-scalar ratio, r) has been obtained in this setup. Furthermore, we have considered the consistency relation in this Galileon model and found that the consistency relation of the standard inflation gets modified due to the presence of the terms G_3 and G_4 in our action (2.1). Finally, to study the non-Gaussian feature of the primordial perturbations in our model, we have studied the non-linear theory in details. In order to investigate non-linear perturbation in the setup, one has to expand the action up to the cubic order in perturbations and obtain the three-point correlation functions. Hence, using the interacting picture we have calculated the three-point correlation functions and the nonlinearity parameter, f_{NL} , in this generalized model. After introducing the shape

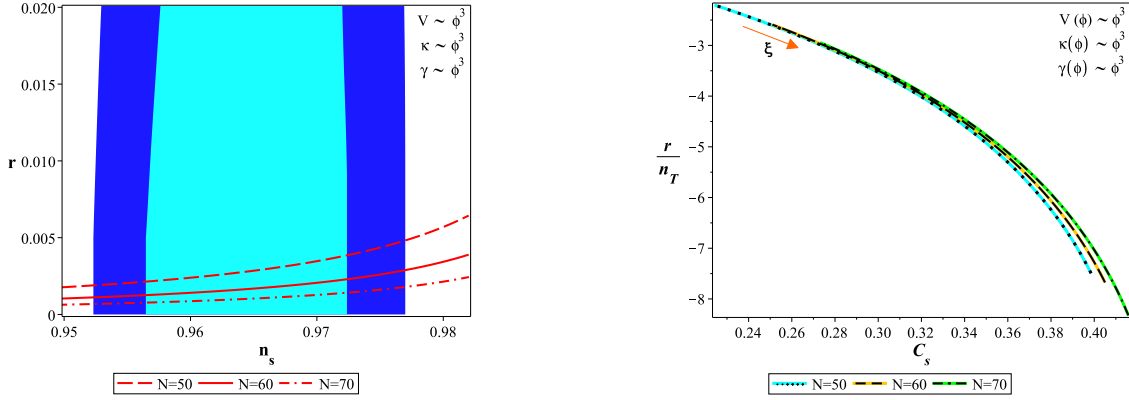


Figure 3. In the left panel the tensor-to-scalar ratio versus the scalar spectral index in the background of the Planck2015 TT, TE, EE+lowP data for a Galileon inflationary model is plotted with $m = 3$, which corresponds to potential of the form $V \sim \phi^3$ and also $\mathcal{K} \sim \phi^3$ and $\gamma \sim \phi^3$ for $N = 50, 60$ and 70 . Using the constraints obtained for ξ in this case, the ratio $\frac{r}{n_T}$ for the model is plotted as a function of the sound speed, c_s , for each e-folds number in the right panel. This figure shows deviation from the standard consistency relation $r = -8n_T$.

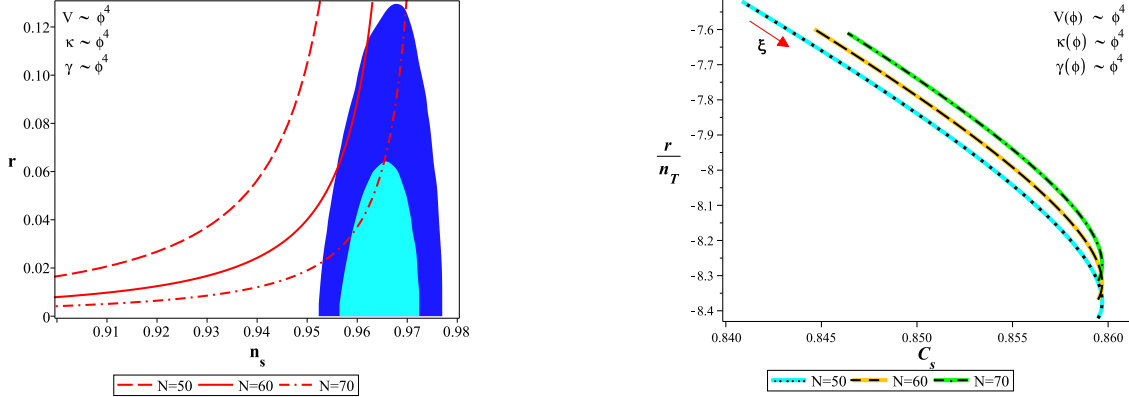


Figure 4. In the left panel the tensor-to-scalar ratio versus the scalar spectral index in the background of the Planck2015 TT, TE, EE+lowP data for a Galileon inflationary model is plotted with $m = 4$, which corresponds to potential of the form $V \sim \phi^4$ and also $\mathcal{K} \sim \phi^4$ and $\gamma \sim \phi^4$ for $N = 50, 60$ and 70 . Using the constraints obtained for ξ in this case, the ratio $\frac{r}{n_T}$ for the model is plotted as a function of the sound speed, c_s , for each e-folds number in the right panel. This figure shows *slight* deviation from the standard consistency relation $r = -8n_T$.

functions as Z_*^{equil} and Z_*^{ortho} , we have derived the amplitude of non-Gaussianity in both equilateral and orthogonal configurations. We have focused our attention on the equilateral limit ($k_1 = k_2 = k_3$), in which, both the equilateral and orthogonal configurations have peak.

After computing the main perturbation parameters, since the viability of an inflationary model depends on its perturbation parameters consistency with observational data, we have found some observational constraints on the parameters space of the inflationary model at hand. To this end, we have specified general functions of the scalar field in the model as $V(\phi) \sim \phi^m$, $\mathcal{K} \sim \phi^m$ and $\gamma \sim \phi^m$ with $m = 1, 2, 3, 4$. In each case the behavior of the tensor-to-scalar ratio versus the scalar spectral index for $N = 50, 60$, and 70 is depicted in Figs. 1- 4. The results are summarized in Table 1, which shows that for some ranges of the non-minimal coupling parameter, ξ , the Galileon inflation is in a good agreement with recent observation. Furthermore, an important aspect of our study is the violation of the standard slow-roll consistency relation of the single-field inflation, $r = -8n_T$. Although we have

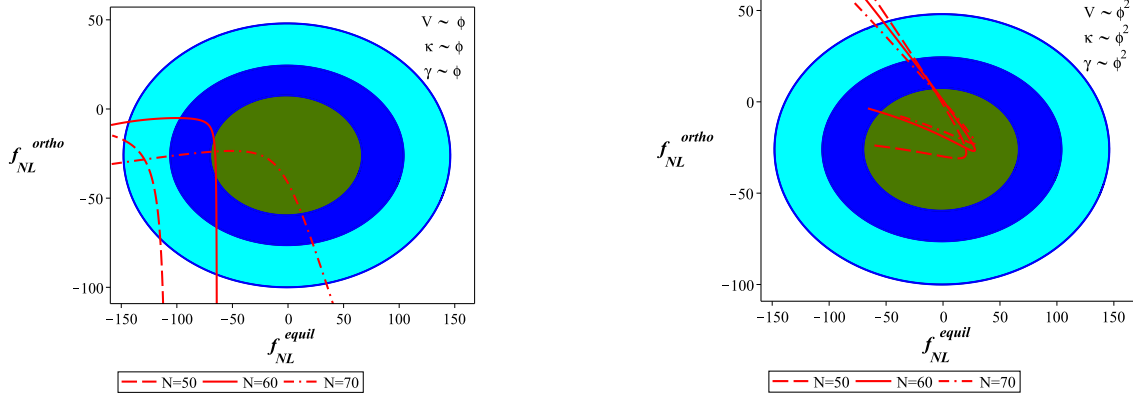


Figure 5. The amplitude of the orthogonal configuration of the non-Gaussianity versus the amplitude of the equilateral configuration for a Galileon inflationary model, in the background of Planck2015 TTT, EEE, TTE and EET data. The left panel is plotted for a linear potential with $m = 1$ while the right panel is plotted for a quadratic potential with $m = 2$. The figures are plotted in the background of Planck2015 TTT, EEE, TTE and EET joint dataset and for $N = 50, 60$ and 70 .

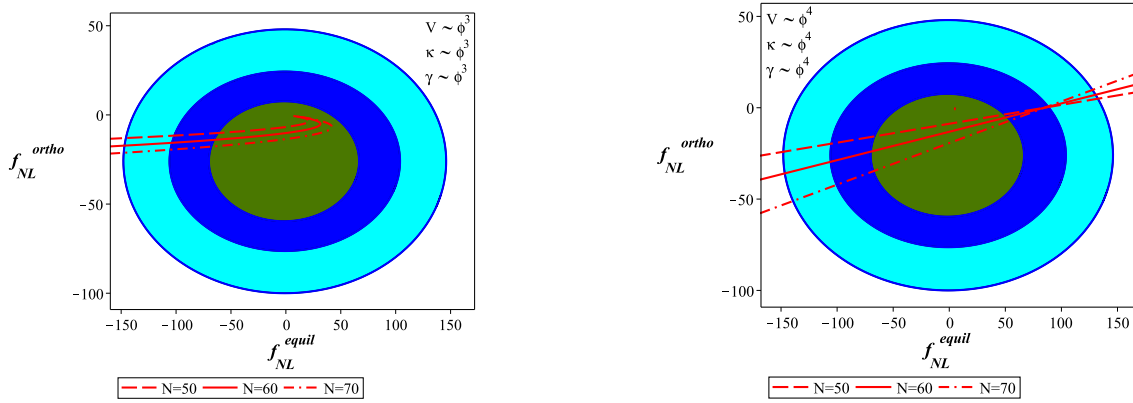


Figure 6. The amplitude of the orthogonal configuration of the non-Gaussianity versus the amplitude of the equilateral configuration for a Galileon inflationary model, in the background of Planck2015 TTT, EEE, TTE and EET data. The right panel is plotted for a cubic potential with $m = 3$ while the right panel is plotted for a quartic potential with $m = 4$. The figures are plotted in the background of Planck2015 TTT, EEE, TTE and EET joint dataset and for $N = 50, 60$ and 70 .

observed from (3.29) that the standard consistency relation of the single-field inflation is in general violated in this setup, however, we emphasize that, by choosing a quadratic form for the scalar field functions, $V \sim \phi^2$, $\mathcal{K} \sim \phi^2$ and $\gamma \sim \phi^2$, we found a *slight* deviation from the standard consistency relation which seems not to be enough significant to be detected confidently.

Moreover, the non-Gaussianity feature of the primordial perturbations have been analyzed numerically by studying the behavior of the orthogonal configuration versus the equilateral configuration at the equilateral limit, $k_1 = k_2 = k_3$, and in the background of the Planck2015 TTT, EEE, TTE and EET data. The results are shown in figures (5) and (6). Also the related constraints are presented in Table 2. As another important result, we have shown that this Galileon inflationary model allows to have large non-Gaussianity in some ranges of the non-minimal coupling parameter that would be observable by future improvements in experiments.

Appendix A: Expansion of the action up to the second order

By replacing the perturbed metric (3.3) in the action and expanding the action up to the second order in perturbations, the following expression will be obtained

$$\begin{aligned}
S_2 = \int dt d^3x a^3 & \left[-3 \left(M_{pl}^2 + \xi \phi^2 - 3 \frac{X^2}{\mu^2} \right) \dot{\Theta}^2 - \frac{2}{a^2} \left(M_{pl}^2 + \xi \phi^2 - 3 \frac{X^2}{\mu^2} \right) \mathcal{B} \partial^2 \Theta + \right. \\
& \frac{1}{a^2} \left(\left(2M_{pl}^2 + 2\xi \phi^2 - 6 \frac{X^2}{\mu^2} \right) \dot{\Theta} - \left(2HM_{pl}^2 + 2H \left(\xi \phi^2 + \frac{X^2}{\mu^2} \right) + 2\gamma X \dot{\phi} - 32H \frac{X^2}{\mu^2} + 2\xi \phi \dot{\phi} \right) \mathcal{B} \right) \partial^2 \zeta + \\
& \left(6H \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right) + 6\gamma X \dot{\phi} - 96H \frac{X^2}{\mu^2} + 6\xi \phi \dot{\phi} \right) \mathcal{B} \dot{\Theta} + \left(\mathcal{K}X - 3H^2 \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right) - \right. \\
& \left. \left. 12\gamma H X \dot{\phi} + 4\gamma_{,\phi} X^2 + 138H^2 \frac{X^2}{\mu^2} - 6\xi H \phi \dot{\phi} \right) \mathcal{B}^2 + \frac{1}{a^2} \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right) (\partial \Theta)^2 \right]. \tag{6.1}
\end{aligned}$$

By varying this second order action with respect to the lapse function and the shift vector, the equations of motion for \mathcal{B} and ζ yield the constraints

$$\mathcal{B} = \left(\frac{M_{pl}^2 + \xi \phi^2 - 3 \frac{X^2}{\mu^2}}{H \left(M_{pl}^2 + \xi \phi^2 - 15 \frac{X^2}{\mu^2} \right) + \gamma X \dot{\phi} + \xi \phi \dot{\phi}} \right) \dot{\Theta}, \tag{6.2}$$

and

$$\begin{aligned}
\frac{1}{a^2} \partial^2 \zeta = 3\dot{\Theta} + & \left[-3H^2 \left(M_{pl}^2 + \xi \phi^2 - 45 \frac{X^2}{\mu^2} \right) + \mathcal{K}X - 12\gamma H X \dot{\phi} + 4\gamma_{,\phi} X^2 - 6\xi H \phi \dot{\phi} \right] \mathcal{B} \times \\
& \left[H \left(M_{pl}^2 + \xi \phi^2 - 15 \frac{X^2}{\mu^2} \right) + \gamma X \dot{\phi} + \xi \phi \dot{\phi} \right]^{-1} - \left[\frac{M_{pl}^2 + \xi \phi^2 - 3 \frac{X^2}{\mu^2}}{H \left(M_{pl}^2 + \xi \phi^2 - 15 \frac{X^2}{\mu^2} \right) + \gamma X \dot{\phi} + \xi \phi \dot{\phi}} \right] \frac{1}{a^2} \partial^2 \Theta. \tag{6.3}
\end{aligned}$$

Finally, substituting the equation of motion (6.2) in the second order action and taking some integrations by parts, we find the quadratic action expressed in (3.4).

Appendix B: Expansion of the action up to the third order

The third order action can be obtained as follows

$$\begin{aligned}
S_3 = \int dt d^3x \Bigg\{ & a^3 \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right) \left[-\frac{3\mathcal{E}_s}{c_s^2} \left(\frac{1}{c_s^2} - 1 \right) + \frac{\mathcal{E}_s}{c_s^4} \left(\mathcal{E}_s - \frac{\dot{\mathcal{E}}_s}{H\mathcal{E}_s} \right) + \right. \\
& \left(\frac{\mathcal{E}_s}{c_s^4 \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right)} \right) \left(\frac{4\gamma X \dot{\phi}}{H} - 44 \frac{X^2}{\mu^2} \right) \Bigg] \Theta \dot{\Theta}^2 + a \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right) \left[\mathcal{E}_s \left(\frac{1}{c_s^2} - 1 \right) + \right. \\
& \left. \frac{\mathcal{E}_s}{c_s^2} \left(\mathcal{E}_s + \frac{\dot{\mathcal{E}}_s}{H\mathcal{E}_s} + 4 \frac{X^2}{\mu^2 \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right)} - \frac{2\dot{c}_s}{Hc_s} \right) \right] \Theta (\partial\Theta)^2 + a^3 \left[\frac{\mathcal{E}_s}{M_{pl} H c_s^2} \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma} + \Gamma \right) \right] \times \\
& \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right) \dot{\Theta}^3 - \frac{2a^3 \mathcal{E}_s}{c_s^2} \dot{\Theta} (\partial_i \Theta) (\partial_i \mathcal{Q}) + \left(\frac{a^3}{4 \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right)} \right) \times \\
& \left[\mathcal{E}_s + \frac{4\gamma X \dot{\phi} - 8H \frac{X^2}{\mu^2}}{H \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right)} \right] (\partial^2 \Theta) (\partial \mathcal{Q}) - \frac{2a M_{pl}}{H^2 \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right)} \left(4H \frac{X^2}{\mu^2} - \gamma X \dot{\phi} \right) \times \\
& \left[\partial^2 \Theta (\partial_i \Theta) (\partial_j \mathcal{Q}) - \Theta \partial_i \partial_j (\partial_i \Theta) (\partial_j \mathcal{Q}) \right] - \frac{a}{H^3} \left[12H \frac{X^2}{\mu^2} - 2\gamma X \dot{\phi} \right] \dot{\Theta}^2 (\partial^2 \Theta) + \\
& \left. \frac{2}{3a} \left(\frac{\gamma X \dot{\phi} - 6H \frac{X^2}{\mu^2}}{H^3} \right) \left[\partial^2 \Theta (\partial\Theta)^2 - \Theta \partial_i \partial_j (\partial_i \Theta) (\partial_j \Theta) \right] \right\}, \tag{6.4}
\end{aligned}$$

where the parameters λ , Σ and Γ are defined by the following expressions

$$\lambda = \frac{1}{3M_{pl}^4} \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right)^2 \left[4\gamma_{,\phi} X^2 - 3H\gamma X \dot{\phi} + 54H^2 \frac{X^2}{\mu^2} \right], \tag{6.5}$$

$$\begin{aligned}
\Sigma = \frac{M_{pl}^2 + \xi \phi^2 - 3 \frac{X^2}{\mu^2}}{M_{pl}^4} \Bigg[& 3 \left(H M_{pl}^2 + H \left(\xi \phi^2 + \frac{X^2}{\mu^2} \right) + \gamma X \dot{\phi} - 16H \frac{X^2}{\mu^2} + \xi \phi \dot{\phi} \right)^2 + \\
& \left(-3M_{pl}^2 H^2 - 3H^2 \left(\xi \phi^2 + \frac{X^2}{\mu^2} \right) + \mathcal{K}X - 12\gamma X H \dot{\phi} + 4\gamma_{,\phi} X^2 + 138H^2 \frac{X^2}{\mu^2} - 6\xi H \phi \dot{\phi} \right) \times \\
& \left. \left(M_{pl}^2 + \xi \phi^2 - 3 \frac{X^2}{\mu^2} \right) \right], \tag{6.6}
\end{aligned}$$

and

$$\begin{aligned}
\Gamma = \frac{1}{M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2}} \Bigg\{ & \frac{\gamma X \dot{\phi}}{H} \left(3 - \frac{1}{c_s^2} \right) - 12 \frac{X^2}{\mu^2} \left(4 - \frac{1}{c_s^2} \right) + \frac{\xi \phi \dot{\phi}}{H} \left(1 - \frac{1}{c_s^2} \right) - \left[\left(\frac{\gamma X \dot{\phi}}{H} \right)^2 + \right. \\
& \left. \frac{\gamma X \dot{\phi}}{H} \left(\frac{\xi \phi \dot{\phi}}{H} - 30 \frac{X^2}{\mu^2} \right) + 136 \frac{X^4}{H\mu^4} - \left(\frac{X^2}{H\mu^2} \right) \left(\xi \phi \dot{\phi} - 30H \frac{X^2}{\mu^2} \right) \right] \left(\frac{6c_s^2}{\mathcal{E}_s \left(M_{pl}^2 + \xi \phi^2 + \frac{X^2}{\mu^2} \right)} \right) \Bigg\}. \tag{6.7}
\end{aligned}$$

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